EXERCISE NO: 5.1

Question 1:

In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Solution 1:

(i) Given :, Fare of First Km = Rs.15

Fare for each Additional Km = Rs. 8

Hence , Taxi fare for 1st km = 15

Taxi fare for first 2 km = 15 + 8 = 23

Taxi fare for first 3 km = 23 + 8 = 31

Taxi fare for first 4 km = 31 + 8 = 39

Hence the Series formed is 15, 23, 31, 39

Since here the terms continuously increases by the same number 8, the above list forms an A.P.

(ii) Let the initial volume of air in a cylinder be V lit. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in the cylinder at a time.

In other words, after every stroke, only $1-\frac{1}{4}=\frac{3}{4}$ th part of air will remain.

Hence the series can be written as below

$$v, \frac{3v}{4}, \frac{3v}{4}-\left(\frac{3v}{4}\right)\left(\frac{1}{4}\right)....$$

$$=v, \frac{3v}{4}, \frac{12v-3v}{16}...$$

$$=v, \frac{3v}{4}, \frac{9v}{16}$$

Now Common difference,

d = Second Term – First Term $=\frac{3v}{4}-v=\frac{-v}{4}$

d = Third Term – Second Term $=\frac{9v}{16}-\frac{3v}{4}=\frac{9v-12v}{16}=\frac{-3v}{16}$

From the above values of "d", common difference is not same between the series. Hence this is not an AP series

(iii) Cost of digging for first metre = 150

Cost of digging for first 2 metres = 150 + 50 = 200

Cost of digging for first 3 metres = 200 + 50 = 250

Cost of digging for first 4 metres = 250 + 50 = 300

Clearly, 150, 200, 250, 300 … forms an A.P. because the common difference between the each term in the series above is constant and is 50.

(iv) We know that if Rs P is deposited at *r*% compound interest per annum for *n* years, our money will be $P\left(1+\frac{r}{100}\right)^{n}$ after *n* years.

Therefore, after every year, our money will be

10000,

$$10000, 10000\left(1+\frac{8}{100}\right), 10000\left(1+\frac{8}{100}\right)^{2}, 10000\left(1+\frac{8}{100}\right)^{3}, 10000\left(1+\frac{8}{100}\right)^{4} ....$$

$$10000, 10000\times\frac{108}{100}, 10000\times\frac{108}{100}\times\frac{108}{100}, 10000\times\frac{108}{100}\times\frac{108}{100}\times\frac{108}{100} .....$$

$$10000, 10800, 11664, ...$$

Now Common difference,

d = Second Term – First Term = 10800- 10000= 800

d = Third Term – Second Term = 11664- 10800 =

From the above values of "d", common difference is not same between the series. Hence this is not an AP series.

Question 2:

Write first four terms of the A.P. when the first term *a* and the common difference *d* are given as follows

(i) $a = 10$, $d = 10$

(ii) $a = -2$, $d = 0$

(iii) $a = 4$, $d = -3$

(iv) $a = -1$ $d = 1/2$
(v) $a = -1.25$, $d = -0.25$

Solution 2:

(i) $a = 10$, $d = 10$

Let the series be $a_1, a_2, a_3, a_4 \ldots$

$a_1 = a = 10$
$a_2 = a_1 + d = 10 + 10 = 20$
$a_3 = a_2 + d = 20 + 10 = 30$
$a_4 = a_3 + d = 30 + 10 = 40$
$a_5 = a_4 + d = 40 + 10 = 50$

Therefore, the series will be 10, 20, 30, 40, 50 …

Hence, First four terms of this A.P. will be 10, 20, 30, and 40.

(ii) $a = -2$, $d = 0$

Let the series be $a_1, a_2, a_3, a_4 \ldots$

$a_1 = a = -2$
$a_2 = a_1 + d = -2 + 0 = -2$
$a_3 = a_2 + d = -2 + 0 = -2$
$a_4 = a_3 + d = -2 + 0 = -2$

Therefore, the series will be −2, −2, −2, −2 …

Hence ,First four terms of this A.P. will be −2, −2, −2 and −2.

(iii) $a = 4$, $d = -3$

Let the series be $a_1, a_2, a_3, a_4 \ldots$

$a_1 = a = 4$
$a_2 = a_1 + d = 4 - 3 = 1$
$a_3 = a_2 + d = 1 - 3 = -2$
$a_4 = a_3 + d = -2 - 3 = -5$

Therefore, the series will be 4, 1, −2 −5 …

Hence ,First four terms of this A.P. will be 4, 1, −2 and −5.

(iv) $a = -1$, $d = 1/2$

Let the series be $a_1, a_2, a_3, a_4 \ldots$

$a_1 = a = -1$

$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$

$a_3 = a_2 + d = -1 + \frac{1}{2} + \frac{1}{2} = 0$

$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$

Clearly, the series will be $-1, -\frac{1}{2}, 0, \frac{1}{2}$............

Hence, First four terms of this A.P. will be $-1, -\frac{1}{2}, 0 \text{and} \frac{1}{2}$

(v) $a = -1.25$, $d = -0.25$

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a = -1.25$

$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$

$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$

$a_4 = a + d = -1.75 - 0.25 = -2.00$

Clearly, the series will be 1.25, −1.50, −1.75, −2.00 ……..

Hence , first four terms of this A.P. will be −1.25, −1.50, −1.75 and −2.00.

Question 3:

For the following A.P.s, write the first term and the common difference.

(i) 3, 1, − 1, − 3 …

(ii) − 5, − 1, 3, 7 …

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$......

(iv) 0.6, 1.7, 2.8, 3.9 …

Solution 3:

(i) The given AP is 3, 1, −1, −3 …

Here, first term, $a = 3$

Common difference, d = Second term − First term

= 1 – 3

= −2

(ii) The given AP is −5, −1, 3, 7 …

Here, first term, $a = -5$

Common difference, d = Second term − First term

= (−1) − (−5)

= − 1 + 5

$= 4$

(iii) The given AP is $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}......$

Here, first term, $a = \frac{1}{3}$

Common difference, d = Second term − First term

$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) The given AP is 0.6, 1.7, 2.8, 3.9 …

Here, first term, $a = 0.6$

Common difference, d = Second term − First term

$= 1.7 - 0.6$

$= 1.1$

Question 4: Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.

$(i)\, 2, 4, 8, 16...$

$(ii)\, 2, \frac{5}{2}, 3, \frac{7}{2}, ...$

$(iii)\, -1.2, -3.2, -5.2, -7.2...$

$(iv)\, -10, -6, -2, 2...$

$(v)\, 3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}...$

$(vi)\, 0.2, 0.22, 0.222, 0.2222.....$

$(vii)\, 0, -4, -8, -12....$

$(viii)\, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}.....$

Solution 4:

(i) The given Series is 2, 4,8 ,16….

First Term, $a_1 = 2$

Second Term, $a_2 = 4$

Third Term, $a_3 = 8$

Common difference d,= $a_2 – a_1$ = 4 –2 =2

= $a_3 – a_2$ = 8 – 4 =4

As ,a_2 - $a_1 \neq a_3 – a_2$

Hence the given series does not form an AP

(ii) The given Series is 2, 5/2, 3, 7/2 ....

First Term, a_1= 2

Second Term, a_2 = 5/2

Third Term, a_3 = 3

Fourth Term, a_4 = 7/2

Fifth Term, a_5 = ?

Sixth Term, a_6 =?

Seventh Term, a_7 = ?

Common difference d,= a_2 - a_1 = 5/2-2 =1/2

= $a_3 – a_2$ = 3- 5/4 =1/2

As a_2 - a_1 = a3 – a2 , given series form an AP.

Fifth Term, a_5 = 7/2 +1/2 =

Sixth Term, a_6 = 4 + 1 /2 = 9/2

Seventh Term, a_7 = 9/2 + 1 /2 = 5

(iii) The given Series is -1.2, - 3.2, -5.2, -7.2 …

First Term, a_1= 2

Second Term, a_2 = 4

Third Term, a_3 = 8

Fourth Term, a_4 = -7.2

Fifth Term, a_5 = ?

Sixth Term, a_6 =?

Seventh Term, a_7 = ?

Common difference d,= a_2 - a_1 = (-3.2) - (-1 .2) = -2

= a_3 – a_2 = (-5.2) - (-3.2) = -2

= a_4 – a_3 = (-7.2) - (-5.2) = -2

As ,a_2 - a_1 = a_3 – a_2 = a_4 – a_3 they form an AP

Fifth Term, a_5 = - 7.2 - 2 = - 9.2

Sixth Term, a_6 = - 9.2 - 2 = - 11 .2

Seventh Term, a_7 = - 11 .2 - 2 = - 1 3.2

(iv) The given Series is -10, - 6, - 2, 2 …

First Term, a_1= -10

Second Term, a_2 = -6

Third Term, a_3 = -2

Fourth Term, a_4 = -2

Common difference d,= a_2 - a_1 = (-6) - (-10) = 4

= a_3 – a_2 = (-2) - (-6) = 4

= a_4 – a_3 = (2) - (-2) = 4

As ,a_2 - a_1 = a_3 – a_2 = a_4 – a_3 they form an AP

Fifth Term, a_5 = 2 + 4 = 6

Sixth Term, a_6 = 6 + 4 = 10

Seventh Term, a_7 = 10 + 4 = 14

(v) The given Series is 3, 3 + √2, 3 + 2√2, 3 + 3√2

First Term, a_1= 3

Second Term, $a_2 = 3 + \sqrt{2}$

Third Term, $a_3 = 3 + 2\sqrt{2}$

Fourth Term, $a_4 = 3 + 3\sqrt{2}$

Common difference d,$= a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

$= a_3 – a_2 = (3 + 2\sqrt{2}) - (3 + \sqrt{2}) = \sqrt{2}$

$= a_4 – a3 = (3 + 3\sqrt{2}) - (3 + 2\sqrt{2}) = \sqrt{2}$

As ,$a_2 - a_1 = a_3 – a_2 = a_4 – a_3$ they form an AP

Fifth Term, $a_5 = (3 + \sqrt{2}) + \sqrt{2} = 3 + 4\sqrt{2}$

Sixth Term, $a_6 = (3 + 4\sqrt{2}) + \sqrt{2} = 3 + 5\sqrt{2}$

Seventh Term, $a_7 = (3 + 5\sqrt{2}) + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) Te given Series is 0.2, 0.22, 0.222, 0.2222 ….

First Term, $a_1 = 0.2$

Second Term, $a_2 = 0.22$

Third Term, $a_3 = 0.222$

Fourth Term, $a_4 = 0.2222$

Common difference d,$= a_2 - a_1 = 0.22 - 0.2 = 0.02$

$= a_3 – a_2 = 0.222 - 0.22 = 0.002$

$a_4 – a3 = 0.2222 - 0.222 = 0.0002$

As ,$a_2 - a_1 \neq a_3 – a_2 \neq a_4 – a_3$

Hence the given series does not form an AP

(vii) To given Series is 0, -4, -8, -12 …

First Term, $a_1 = 0$

Second Term, $a_2 = -4$

Third Term, $a_3 = -8$

Fourth Term, $a_4 = -12$

Common difference d,= $a_2 - a_1 = (-4) - 0 = -4$

$= a_3 – a_2 = (-8) - (-4) = -4$

$= a_4 – a3 = (-12) - (-8) = -4$

As ,$a_2 - a_1 = a_3 – a_2 = a_4 – a_3$ they form an AP

Fifth Term, $a_5 = -12 - 4 = -16$

Sixth Term, $a_6 = -16 - 4 = -20$

Seventh Term, $a_7 = -20 - 4 = -24$

(viii) To given Series is -1 /2, -1 /2, -1 /2, -1 /2 ....

First Term, $a_1 = -1/2$

Second Term, $a_2 = -1/2$

Third Term, $a_3 = -1/2$

Fourth Term, $a_4 = -1/2$

Common difference d,= $a_2 - a_1 = (-1 /2) - (-1 /2) = 0$

$= a_3 – a_2 = (-1 /2) - (-1 /2) = 0$

$= a_4 – a_3 = (-1 /2) - (-1 /2) = 0$

As ,$a_2 - a_1 = a_3 – a_2 = a_4 – a_3$ they form the AP Series

Fifth Term, $a_5 = (-1 /2) - 0 = -1 /2$

Sixth Term, $a_6 = (-1 /2) - 0 = -1 /2$

Seventh Term, a_7 $(-1 /2) - 0 = -1 /2$

(ix) The gven Series is 1 , 3, 9, 27 …

First Term, $a_1 = 1$

Second Term, $a_2 = 3$

Third Term, $a_3 = 9$

Fourth Term, $a_4 = 27$

Common difference d,= $a_2 - a_1 = = 3 - 1 = 2$

$= a_3 – a_2 = 9 - 3 = 6$

$= a_4 – a_3 = 27 - 9 = 1 8$

As ,$a_2 - a_1 \neq a_3 – a2$

Hence the given series does not form AP

(x) The given Series is a, 2a, 3a, 4a …

First Term, $a_1= a$

Second Term, $a_2 = 2a$

Third Term, $a_3 = 3a$

Fourth Term, $a_4 = 4a$

Common difference d,= $a_2 - a_1 = 2a- a= a$

$= a_3 – a_2 = 3a-2a = a$

$= a_4 – a_3 = 4a-3a =a$

As ,$a_2 - a_1 = a_3 – a_2 = a_4 – a_3$ they form the AP Series

Fifth Term, $a_5 = 4a+ a =5a$

Sixth Term, $a_6 = 5a+a =6a$

Seventh Term, $a_7 = 6a+a =7a$

(xi) The given Series is a, a^2, a^3, a^4 …

First Term, $a_1= a$

Second Term, $a_2 = a^2$

Third Term, $a_3 = a^3$

Fourth Term, $a_3 = a^4$

Common difference d,= $a_2 - a_1 = a^2 – a = a -1$

$= a_3 – a_2 = a^3 - a^2 = a^2 (a-1)$

$= a_4 – a_3 = a^4 – a^3 = a^3)(a-1)$

As ,$a_2 - a_1 \neq a_3 – a_2$,

Hence it is not the AP series

(xii) The given Series is √2, √8, √18, √32 ...

First Term, $a_1 = \sqrt{2}$

Second Term, $a_2 = \sqrt{8}$

Third Term, $a_3 = \sqrt{18}$

Fourth Term, $a_4 = \sqrt{32}$

Common difference d = $a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$= a_3 – a_2 = \sqrt{1\ 8} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$= a_4 – a_3 = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

As ,$a_2 - a_1 = a_3 – a_2 = a_4 – a_3$ they form the AP Series

Fifth Term, $a_5 = \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

Sixth Term, $a_6 = 5\sqrt{2} +\sqrt{2} = 6\sqrt{2} = \sqrt{72}$

Seventh Term, $a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) The given Series is √3, √6, √9, √1 2 ...

First Term, $a_1 = \sqrt{3}$

Second Term, $a_2 = \sqrt{6}$

Third Term, $a_3 = \sqrt{9}$

Fourth Term, $a_4 = \sqrt{12}$

Common difference d,$= a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3} \times 2 - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$

$= a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}(\sqrt{3} - \sqrt{2})$

$= a_4 - a_3 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}(\sqrt{3} - \sqrt{2})$

As ,$a_2 - a_1 \neq a_3 - a_2 \neq a_4 - a_3$,

Hence they do not form the AP series

(xiv) The given Series is $1^2, 3^2, 5^2, 7^2 \ldots$

First Term, $a_1 = 1^2$

Second Term, $a_2 = 3^2$

Third Term, $a_3 = 5^2$

Fourth Term, $a_4 = 7^2$

Common difference d,$= a_2 - a_1 = 3^2 - 1^2 = 8$

$= a_3 - a_2 = 25 - 9 = 16$

$= a_4 - a_3 = 49 - 25 = 24$

As ,$a_2 - a_1 \neq a_3 - a_2 \neq a_4 - a_3$,

Hence they do not form the AP Series

(xv) The given Series is 1^2, 5^2, 7^2, 73 …

First Term, $a_1 = 1^2 = 1$

Second Term, $a_2 = 5^2 = 25$

Third Term, $a_3 = 7^2 = 49$

Fourth Term, $a_4 = 73$

Common difference d,$= a_2 - a_1 = 25 - 1 = 24$

$= a_3 – a_2 = 49 – 25 = 24$

$= a_4 – a_3 = 73 - 49 = 24$

As ,$a_2 - a_1 = a_3 – a_2 = a_4 – a_3$ they form the AP Series

Fifth Term, $a_5 = 73 + 24 = 97$

Sixth Term, $a_6 = 97 + 24 = 121$

Seventh Term, $a_7 = 121 + 24 = 145$

EXERCISE NO: 5.2

Question 1:

Fill in the blanks in the following table, given that *a* is the first term, *d* the common difference and a_n the n^{th} term of the A.P.

	a	d	n	a_n
I	7	3	8	….
II	−18	…..	10	0
III	…..	−3	18	−5
IV	−18.9	2.5	…..	3.6
V	3.5	0	105	…..

Solution 1:

I. Given,

- *First Term*, $a = 7$,
- *Common Difference*, $d = 3$,
- *Number of Terms*, $n = 8$,
- n^{th} *term of an AP Series*, $a_n = ?$

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1)\,d$

$= 7 + (8 - 1)\,3$ *(By Substituting)*

$= 7 + (7)\,3$

$= 7 + 21 = 28$

Hence, $a_n = 28$

II. Given,

- *First Term*, $a = -18$,
- n^{th} *term of an AP Series*, $a_n = 0$,
- Number of terms, $n = 10$,
- Common Difference, $d = ?$

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1)\,d$

$0 = -18 + (10 - 1)\,d$ *(By Substituting)*

$18 = 9d$

$9d = 18$

$d = \frac{18}{9} = 2$ *(By Transposing)*

Hence, common difference, $d = 2$

III. Given,

- Common Difference, $d = -3$,
- Number of terms, $n = 18$,
- Nth term of the AP Series, $a_n = -5$
- First Term, a=?
- We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1)\, d$

$-5 = a + (18 - 1)(-3)$ (By Substituting)

$-5 = a + (17)(-3)$

$-5 = a - 51$

$a - 51 = -5$

$a = 51 - 5$

Hence, $a = 46$

IV. Given,

- First Term, $a = -18.9$,
- Common Difference, $d = 2.5$,
- Nth term of the AP Series, $a_n = 3.6$,
- Number of terms, $n = ?$

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1)\, d$

$3.6 = -18.9 + (n - 1)\, 2.5$ (By Substituting)

$3.6 + 18.9 = (n - 1)\, 2.5$

$22.5 = (n - 1)\, 2.5$

$(n-1) = \frac{22.5}{2.5}$ (By transposing)

$n-1=9$

$n = 10$

Hence, $n = 10$

V. **Given,**

- First Term, a = 3.5,
- Common Difference, d = 0,
- Number of terms, $n = 105$,
- Nth term of AP Series, $a_n = ?$

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1)\, d$

$a_n = 3.5 + (105 - 1)\, 0$ (By Substituting)

$a_n = 3.5 + 104 \times 0$

$a_n = 3.5$

Hence, $a_n = 3.5$

Question 2:

Choose the correct choice in the following and justify

I. 30^{th} term of the A.P 10, 7, 4,…, is

A. 97 B. 77 C. − 77 D. – 87

II. 11^{th} term of the A.P. is $-3, \frac{-1}{2}, 2......$

A. 28 B. 22 C. − 38 D. $-48\frac{1}{2}$

Solution 2:

I. Given,

- A.P. Series is 10, 7, 4, …
- First term, $a_1 = 10$
- Second term $a_2 = 7$
- Number of terms, n = 30
- Common difference, $d = a_2 - a_1 = 7 - 10 = -3$
- *30^{th} Term* $, a_{30} = ?$

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1)\, d$

$a_{30} = 10 + (30 - 1)(-3)$ (By Substituting)

$a_{30} = 10 + (29)(-3)$

$a_{30} = 10 - 87 = -77$

Hence, the correct answer is **C**.

II. Given that,

- A.P. Series is $-3, \frac{-1}{2}, 2......$
- First term $a = -3$
- Second Term = − ½
- Nth term of the AP Series, $a_n = ?$
- Common difference, $d = a_2 - a_1$

$$= -\frac{1}{2} - (-3)$$

$$= -\frac{1}{2} + 3 = \frac{5}{2}$$

We know that the n^{th} term of an A.P. Series,

$$a_n = a + (n-1)d$$
$$a_n = -3 + (11-1)\left(\frac{5}{2}\right)$$
$$a_n = -3 + (10)\left(\frac{5}{2}\right)$$
$$a_n = -3 + 25$$
$$a_n = 22$$
Hence, the answer is B.

Question 3:

In the following APs find the missing term in the boxes

I. 2, □, 26

II. □, 13, □, 3

III. 5, □, □, $9\frac{1}{2}$

IV. −4, □, □, □, □, 6

V. □, 38, □, □, □, −22

Solution 3:

I. 2, □, 26

Given,

- a=2
- d=?

Let the above term be a, a+d and a+2d

Hence Third Term, a+2d = 26

By Substituting the value of a in the above,

26 = 2 + 2*d*

24 = 2*d*

d = 12

Hence Missing term, a+d = 2+12 = 14

II. □,13, □, 3

For this A.P.,

Given

- *Second Term,* a_2=13
- *Fourth Term,* a_4 = 3
- *First Term,a =?*

- *Fourth Term*,a_3 =?

We know that the n[th] term of an A.P. Series,

$a_n = a + (n - 1)\, d$

$a_2 = a + (2 - 1)\, d$

$13 = a + d$ …….Equation (I)

$a_4 = a + (4 - 1)\, d$

$3 = a + 3d$ ……..Equation (II)

On subtracting Equation (I) from Equation (II), we obtain

$a+d - (a + 3d) = 13-3$

$a + d - a - 3d = 10$

$-2d = 10$

$d = -5$

By Substituting $d = -5$ in equation (I), we obtain

$13 = a + (-5)$

$a = 18$

$a_3 = 18 + (3 - 1)(-5)$

$= 18 + 2(-5) = 18 - 10 = 8$

Therefore, the missing terms are 18 and 8 respectively.

III. 5, ☐ , ☐ , $9\frac{1}{2}$

For this A.P.,

- First Term , a= 5
- Fourth term $a_4 = 9\frac{1}{2} = \frac{19}{2}$

We know that the n[th] term of an A.P. Series,

$a_n = a + (n+1)d$

$a_4 = a + (4-1)d$

$\frac{19}{2} = 5 + 3d$

$\frac{19}{2} - 5 = 3d$

$\frac{9}{2} = 3d$

$d = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$

Therefore, the missing terms are $\frac{13}{2}$ and 8 respectively.

IV. −4, □, □, □, □, 6

For this A.P.,

- *First Term,* $a = -4$
- *Sixth Term,* $a_6 = 6$
- *Common Difference,* $d = ?$
- Second term $a_2 = ?$
- Third Term, $a_3 = ?$
- Fourth Term, $a_4 = ?$
- Fifth Term, $a_5 = ?$

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1)\, d$

$a_6 = a + (6 - 1)\, d$

$6 = -4 + 5d$ *(By Substitutuing)*

$10 = 5d$

$d = 2$

Hence,

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2\,(2) = 0$

$a_4 = a + 3d = -4 + 3\,(2) = 2$

$a_5 = a + 4d = -4 + 4\,(2) = 4$

Therefore, the missing terms are −2, 0, 2, and 4 respectively.

V. □, 38, □, □, □, −22

For this A.P.,

- *Second term,* $a_2 = 38$
- *Sixth Term,* $a_6 = -22$
- *First Term,* $a = ?$
- Third Term, $a_3 = ?$
- Fourth Term, $a_4 = ?$
- Fifth Term, $a_{5=?}$
- Common Difference, d = ?

We know that the n[th] term of an A.P. Series,
$a_n = a + (n - 1)\, d$

$a_2 = a + (2 - 1)\, d$
$38 = a + d$ ………. Equation (1)

$a_6 = a + (6 - 1)\, d$
$-22 = a + 5d$ ……….Equation (2)

On subtracting equation (1) from *Equation* (2), we obtain

$-22 - 38 = 4d$
$-60 = 4d$
$d = -15$

Hence,
$a = a2 - d = 38 - (-15) = 53$
$a_3 = a + 2d = 53 + 2\,(-15) = 23$
$a_4 = a + 3d = 53 + 3\,(-15) = 8$
$a_5 = a + 4d = 53 + 4\,(-15) = -7$
Therefore, the missing terms are 53, 23, 8, and −7 respectively

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Question 4:
Which term of the A.P. 3, 8, 13, 18, … is 78?

Solution 4:
Given Series, 3, 8, 13, 18, …
For this A.P.,

- *First Term,* $a = 3$
- *Common difference,* $d = a_2 - a_1 = 8 - 3 = 5$
- *n*th term of this A.P. $a_n = 78$

- Sixteenth term, $a_{16} = ?$

We know that for the AP series nth term,
$a_n = a + (n - 1)\, d$
$78 = 3 + (n - 1)\, 5$ (By Substituting)
$75 = (n - 1)\, 5$
$(n - 1) = 15$
$n = 16$
Hence, 16^{th} term of this A.P. is 78.

Question 5:
Find the number of terms in each of the following A.P.

I. 7, 13, 19, …, 205
II. $18, 15\frac{1}{2}, 13 ...., -47$

Solution 5:

I. 7, 13, 19, …, 205

For this A.P.,

- *First term, $a = 7$*
- nth term of the AP series, $a_n = 205$
- *Common Difference, $d = a_2 - a_1 = 13 - 7 = 6$*
- *Number of terms, $n = ?$*

We know that the n^{th} term of an A.P. Series,
$a_n = a + (n - 1)\, d$

Therefore, $205 = 7 + (n - 1)\, 6$
$198 = (n - 1)\, 6$
$33 = (n - 1)$
$n = 34$
Therefore, this given series has 34 terms in it.

II. $18, 15\frac{1}{2}, 13 ...., -47$

For this A.P.,

- First Term, a=18
- Common Difference d $= a_2 - a_1$

$$15\frac{1}{2}-18=\frac{31}{2}-18$$
$$=\frac{31-36}{2}$$
$$=\frac{-5}{2}$$

nth term of AP series, $a_n = -47$

We know that the n^{th} term of an A.P. Series,

$$a_n = a+(n-1)d$$
$$-47 = 18+(n-1)\left(-\frac{5}{2}\right)$$
$$-47-18=(n-1)\left(-\frac{5}{2}\right)$$
$$-65=(n-1)\left(-\frac{5}{2}\right)$$
$$(n-1)=\frac{-130}{-5}$$
$$(n-1)=26$$
$$n=27$$

Therefore, this given A.P. has 27 terms in it.

Question 6:

Check whether − 150 is a term of the A.P. 11, 8, 5, 2, …

Solution 6:

For this A.P.,

- *First term, a* = 11
- *Common difference,* $d = a_2 - a_1 = 8 - 11 = -3$
- *N th term of AP series,* $a_n = -150$

We know that the n^{th} term of an A.P. Series,

$$a_n = a+(n-1)d$$
$$-150=11+(n-1)(-3)$$
$$-150=11-3n+3$$
$$-164=-3n$$
$$n=\frac{164}{3}$$

Clearly, n is not an integer.
Therefore, −150 is not a term of this A.P.

Question 7:
Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73

Solution 7:
Given that,

- *11^{th} term,* $a_{11} = 38$
- *16^{th} Term,* $a_{16} = 73$
- *Common difference, d=?*
- *31^{st} term,* a_{31} =?

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1)\, d$
$a_{11} = a + (11 - 1)\, d$
$38 = a + 10d$ ……….Equation (1)

Similarly,
$a_{16} = a + (16 - 1)\, d$
$73 = a + 15d$ ………Equation (2)

On subtracting *Equation* (1) from *Equation* (2), we obtain

$A + 15d - (a + 10d) = 73 - 38$
$a - a + 15d - 10d = 35$
$5d = 35$
$d = 7$

By substituting the value of d in Equation (1),

$38 = a + 10 \times (7)$
$38 - 70 = a$
$a = -32$
$a_{31} = a + (31 - 1)\, d$
$= -32 + 30\,(7)$
$= -32 + 210$
$= 178$
Hence, 31^{st} term is 178.

Question 8:

An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term

Solution 8:

Given that,

- *Third term,* $a_3 = 12$
- *50th term,* $a_{50} = 106$
- *Common difference,* $d = ?$
- *20th term,* $a_{29} = ?$

We know that the nth term of an A.P. Series,

$a_n = a + (n - 1)\, d$

$a_3 = a + (3 - 1)\, d$

$12 = a + 2d$ ……. Equation (I)

Similarly, $a_{50} = a + (50 - 1)\, d$

$106 = a + 49d$ ………..Equation (II)

On subtracting *Equation* (I) from *Equation* (II), we obtain

$a + 49d - (a + 2d) = 106 - 12$

$49d - 2d = 94$

$47d = 94$

$d = 2$

By substituting the value of d in Equation (I), we obtain

$12 = a + 2\,(2)$

$a = 12 - 4 = 8$

$a_{29} = a + (29 - 1)\, d$

$a_{29} = 8 + (28)^2$

$a_{29} = 8 + 56 = 64$

Therefore, 29th term is 64.

Question 9:

If the 3rd and the 9th terms of an A.P. are 4 and – 8 respectively. Which term of this A.P. is zero.

Solution 9:

Given that,

- *Third term,* $a_3 = 4$
- 9^{th} *term,* $a_9 = -8$
- *Common difference,* $d = ?$
- *For what value of* n*,* $a_n = 0$?

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1) d$

$a_3 = a + (3 - 1) d$

$4 = a + 2d$ ……..*Equation* (I)

$a_9 = a + (9 - 1) d$

$-8 = a + 8d$ ……..Equation (II)

On subtracting Equation (I) from Equation (II),

$a+2d-(a+8d)= 4-(-8)$

$a+2d-a- 8d= 4+8$

$-6d = 12$

$d = -2$

By Substituting the value of d in Equation (I), we obtain

$4 = a + 2 (-2)$

$4 = a - 4$

$a = 8$

Let nth term of this A.P. be zero

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1) d$

$0 = 8 + (n - 1) (-2)$

$0 = 8 - 2n + 2$

$2n = 10$

$n = 5$

Hence, 5th term of this A.P. is 0.

Question 10:

If 17th term of an A.P. exceeds its 10th term by 7. Find the common difference.

Solution 10:

Given,

- $a_{17} - a_{10} = 7$

- *Common difference, d=?*

We know that nth term of the AP series, For an A.P., $a_n = a + (n – 1) d$

$a_{17} = a + (17 – 1) d$

$a_{17} = a + 16d$

Similarly, $a_{10} = a + 9d$

It is given that

$(a + 16d) – (a + 9d) = 7$

$7d = 7$

$d = 1$

Therefore, the common difference is 1.

Question 11:

Which term of the A.P. 3, 15, 27, 39, … will be 132 more than its 54^{th} term?

Solution 11:

Given A.P. is 3, 15, 27, 39, …

- *First term,* $a = 3$
- *Common difference,* $d = a_2 – a_1 = 15 − 3 = 12$

We know that nth term of the AP series, $a_n = a + (n − 1) d$

$a_{54} = a + (54 − 1) d$

$= 3 + (53) (12)$ (By Substituting)

$= 3 + 636 = 639$

Adding 132 to 54^{th} term we get, 132 + 639 = 771

We have to find the term of this A.P. which is 771.

Let n^{th} term be 771.

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n − 1) d$

$771 = 3 + (n − 1) 12$

$768 = (n − 1) 12$

$(n − 1) = 64$

$n = 65$

Therefore, 65th term was 132 more than 54th term.

Alternatively,

Let *n*th term be 132 more than 54th term.

$n = 54 + \frac{132}{12}$

$= 54 + 11 = 65^{th}$ Term

Question 12:

Two APs have the same common difference. The difference between their 100th term is 100, what is the difference between their 1000th terms?

Solution 12:

Let the first term of these A.P.s be a_1 and b_1 respectively and the Common difference of these A.P's be d.

For first A.P.,

$a_{100} = a_1 + (100 - 1)\, d$

$= a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1)\, d$

$a_{1000} = a_1 + 999d$

For second A.P.,

$b_{100} = b_1 + (100 - 1)\, d$

$= b_1 + 99d$

$b_{1000} = b_1 + (1000 - 1)\, d$

$= b_1 + 999d$

Given that, difference between

100^{th} term of these A.P.s = 100

Thus, we have

$(a_1 + 99d) - (b_1 + 99d) = 100$

$a_1 - b_1 = 100$ ....Equation (1)

Difference between 1000th terms of these A.P.s

$(a_1 + 999d) - (b_1 + 999d) = a_1 - b_1$ ....Equation (2)

From equation (1) & Equation (2),

This difference, $a_1 - b_1 = 100$

Hence, the difference between 1000^{th} terms of these A.P. will be 100.

Question 13:
How many three digit numbers are divisible by 7

Solution 13:
First three-digit number that is divisible by 7 = 105
Next number = 105 + 7 = 112

Therefore, the series becomes 105, 112, 119, …

All are three-digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

When we divide 999 by 7, the remainder will be 5.
Clearly, 999 − 5 = 994 is the maximum possible three-digit number that is divisible by 7.
Hence the final series is as follows:
105, 112, 119, …, 994

Let 994 be the *n*th term of this A.P.
$a = 105$
$d = 7$
$a_n = 994$
$n = ?$

We know that the n^{th} term of an A.P. Series,

$a_n = a + (n - 1) d$
$994 = 105 + (n - 1) 7$
$889 = (n - 1) 7$
$(n - 1) = 127$
$n - 1 = 889/7$
$n = 127 + 1$
$n = 128$

Therefore, 128 three-digit numbers are divisible by 7.

Question 14:
How many multiples of 4 lie between 10 and 250?

Solution 14:

By Observation, First multiple of 4 that is greater than 10 is 12.
Next will be 16.

Therefore, The series will be as follows: 12, 16, 20, 24, …

All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.

When we divide 250 by 4, the remainder will be 2. Therefore, 250 – 2 = 248 is divisible by 4 which is the largest multiple of 4 within 250.

Hence the final series is as follows:
12, 16, 20, 24, …, 248

Let 248 be the *n*th term of this A.P.

We know that the n^{th} term of an A.P. Series,

$$a = 12$$
$$d = 4$$
$$a_n = 248$$
$$a_n = 1 + (n - 1)d$$
$$248 = 12 + (n - 1)4$$
$$\frac{236}{4} = n - 1$$
$$59 = n - 1$$
$$n = 60$$

Therefore, there are 60 multiples of 4 between 10 and 250.

Question 15:
For what value of *n*, are the *n*th terms of two APs 63, 65, 67, and 3, 10, 17, … equal

Solution 15:

If nth terms of the two APs are 63, 65, 67, ... and 3, 10, 17, ... are equal.

Then, 63 + (n – 1) 2 = 3 + (n – 1) 7 ....Equation (1)
[Since In 1st AP, a = 63, d = 65 – 63 = 2

and in 2nd AP , a = 3, d = 10 – 3 = 7]
By Simplifying Equation (1)
7(n – 1) – 2 (n – 1) = 63 – 3
7n−7−2n+2 =60
5n −5 = 60
n−1 = 12
n = 12 + 1 = 13

Hence, the 13th terms of the two given APs are equal.

Question 16:
Determine the A.P. whose third term is 16 and the 7th term exceeds the 5th term by 12.

Solution 16:
Let a be the first term and d the common difference.
Hence from given, $a_3 = 16$ and $a_7 - a_5 = 12$
$a + (3 - 1)\, d = 16$
$a + 2d = 16$ ....Equation (1)

Using $a_7 - a_5 = 12$
$[a + (7 - 1)\, d] - [a + (5 - 1)\, d] = 12$
$(a + 6d) - (a + 4d) = 12$
$2d = 12$
d = 6

By Substituting this in Equation (1), we obtain

$a + 2\,(6) = 16$
$a + 12 = 16$
$a = 4$

Therefore, A.P. will be $4+6, 4+2\times6, 4+3\times6....$

Hence the series will be 4, 10, 16, 22, …

Question 17:
Find the 20th term from the last term of the A.P. 3, 8, 13, …, 253

Solution 17:
Given A.P. is 3, 8, 13, …, 253
From Given,
As the 20^{th} term is considered from last a=253

Common difference , d= 3−8 = −5 (Considered in reverse order)
We know that the n^{th} term of an A.P. Series,
$a_n = a + (n - 1)\, d$
Hence 20^{th} *Term* , $a_{20} = a + (20 - 1)\, d$

$a_{20} = 253 + (20 - 1)(-5)$
$= 253 - 19 \times 5$
$= 253 - 95$
$= 158$

Therefore, 20th term from the last term is 158.

Question 18:
The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Solution 18:
Let a be the first term and d the common difference.

Given, $a_4 + a_8 = 24$
$(a + 3d) + (a + 7d) = 24$
$\Rightarrow 2a + 10d = 24$
$\Rightarrow a + 5d = 12$ ....Equation (1)

Also, $a_6 + a_{10} = 44$
$(a + 5d) + (a + 9d) = 44$
$\Rightarrow 2a + 14d = 44$
$\Rightarrow a + 7d = 22$ ....Equation (2)

On subtracting Equation (1) from (2), we obtain

$a + 5d - (a + 7d) = 12 - 22$
$a - a + 5d - 7d = -10$
$-2d = -10$
$2d = 10$

$d = 5$

By Substituting the value of d=5 in Equation (1), we obtain
$a + 5d = 12$
$a + 5\,(5) = 12$
$a + 25 = 12$
$a = -13$

The first three terms are a, (a + d) and (a + 2d)

Substituting the values of a and d, we get – 13, (– 13 + 5) and (– 13 + 2 × 5)

i.e., – 13, – 8 and – 3

Therefore, the first three terms of this A.P. are −13, −8, and −3.

Question 19:
Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Solution 19:
From the Given Data, incomes received by Subba Rao in the years 1995,1996,1997… are 5000, 5200, 5400, ……… 7,000

From Observation, Common difference, d= 200
$a = 5000$
$d = 200$

Let after n^{th} year, his salary be Rs 7000.
Hence $a_n = Rs.7000$
We know that the n^{th} term of an A.P. Series,
$a_n = a + (n - 1)\,d$

By Substituting above values,

$7000 = 5000 + (n - 1)\,200$
$200(n - 1) = 7000 - 5000$
$200(n-1)=2000$
$(n - 1) = 2000/200 = 10$
$n = 11$

Therefore, in 11th year, his salary will be Rs 7000.

Question 20:

Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the *n*th week, her weekly savings become Rs 20.75, find *n*.

Solution 20:

From the given data, Ramkali's savings in the consecutive weeks are

$$Rs.5, Rs.5+Rs.1.75,$$

$$Rs.5+2\times Rs.1.75, Rs.5+3\times Rs.1.75...$$

Hence n^{th} Term is Rs. 5+(n−1) × Rs. 1.75 = Rs. 20.75

Now from the above we know that

$a = 5$

$d = 1.75$

$a_n = 20.75$

$n = ?$

We know that the n^{th} term of an A.P. Series,

$$a_n = a+(n-1)d$$

$$20.75 = 5+(n-1)1.75$$

$$15.75 = (n-1)1.75$$

$$(n-1) = \frac{15.75}{1.75} = \frac{1575}{175}$$

$$= \frac{63}{7} = 9$$

$n - 1 = 9$

$n = 10$

Hence, *n* is 10.

EXERCISE NO: 5.3

Question 1:

Find the sum of the following APs.

(i) 2, 7, 12 ,…., to 10 terms.

(ii) − 37, − 33, − 29 ,…, to 12 terms

(iii) 0.6, 1.7, 2.8 ,…….., to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$,………, to 11 terms

Solution 1:

(i)2, 7, 12 ,…, to 10 terms

Given,

- *First term, a* = 2
- *Common Difference, d* = $a_2 - a_1$ =7− 2 = 5
- *Number of Terms, n* = 10
- *Sum of n terms,* S_{10} =?

We know that Sum of nth term of AP,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(2) + (10-1)5\right]$$

$$= 5\left[4 + (9)\times(5)\right]$$

$$= 5\left[4 + (9)\times(5)\right]$$

$$= 5 \times 49 = 245$$

(ii) −37, −33, −29 ,…, to 12 terms

Given,

- *First term, a* = −37
- *Common Difference, d* = $a_2 - a_1$ = (−33) − (−37) = − 33 + 37 = 4
- *Number of Terms, n* = 12
- *Sum of n terms,* S_{12} =?

We know that Sum of nth term of AP,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(-37) + (12-1)4\right]$$

$$= 6\left[-74 + 11\times 4\right]$$

$$= 6\left[-74 + 44\right]$$

$$= 6(-30) = -180$$

(iii) 0.6, 1.7, 2.8 ,…, to 100 terms
Given,

- *First term, a* = 0.6
- *Common difference, d* = $a_2 - a_1$ = 1.7 − 0.6 = 1.1
- *Number of Terms, n* = 100
- *Sum of n terms,* S_{100} =?

We know that Sum of n[th] term of AP,,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{100} = \frac{100}{2}\left[2(0.6) + (100-1)1.1\right]$$

$$= 50\left[1.2 + (99)\times(1.1)\right]$$

$$= 6\left[1.2 + 108.9\right]$$

$$= 50\left[110.1\right]$$

$$= 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$,………, to 11 terms for this A.P.,
Given,

- First term, a = 1/15
- Number of Terms, n=11
- Common difference, d= a_2-a_1 = 1/12 – 1/15= (5-4)/60= 1/60
- *Sum of n terms,* S_{11} =?

We know that Sum of nth term of series,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \left(\frac{11}{2}\right)\left(\frac{9}{30}\right) = \frac{33}{20}$$

Question 2:

Find the sums given below

(i) $7 + 10\frac{1}{2} + 14 + \ldots\ldots\ldots\ldots + 84$

(ii) $34 + 32 + 30 + \ldots\ldots\ldots\ldots + 10$

(iii) $-5 + (-8) + (-11) + \ldots\ldots\ldots\ldots + (-230)$

Solution 2:

(i)$7 + 10\frac{1}{2} + 14 + \ldots\ldots\ldots\ldots + 84$

Given,

- *First term, a* = 7
- Common difference, d= $10\frac{1}{2} - 7 = 7/2$
- Number of terms, n =?
- *84th Term, l*= $a_n = 84$
- *Sum of n terms,* S_{23} =?

$l = a_n = a + (n - 1)d$

$$84 = 7 + (n-1)\frac{7}{2}$$

$$77 = (n-1)\frac{7}{2}$$

$22 = n - 1$

Hence, $n = 23$

We know that Sum of n terms,

$$S_n = \frac{n}{2}[a + l]$$

$$S_n = \frac{23}{2}[7 + 84]$$

$$= \frac{23 \times 91}{2} = \frac{2093}{2}$$

$$= 1046\frac{1}{2}$$

(ii) $34 + 32 + 30 + \ldots\ldots\ldots\ldots + 10$

For this A.P.,

Given,

- *First term,a* = 34
- *Common Difference,* $d = a_2 - a_1 = 32 - 34 = -2$
- *Last term,* $l = 10 = a_n$

- Number of terms, n =?
- *Sum of n terms,* S_{10} =?

We know that the Term of AP,

$l = a + (n - 1)\, d$

$10 = 34 + (n - 1)(-2)$

$-24 = (n - 1)(-2)$

$12 = n - 1$

Hence, $n = 13$

Sum of n terms of AP is,

$$S_n = \frac{n}{2}[a + l]$$

$$= \frac{13}{2}[34 + 10]$$

$$= \frac{13 \times 44}{2} = 13 \times 22$$

$$= 286$$

(iii) (−5) + (−8) + (−11) + ………… + (−230)

For this A.P.,

Given,

- *First term,* $a = -5$
- N^{th} *Term,* $l = -230$
- *Common difference,* $d = a_2 - a_1 = (-8) - (-5) = -8 + 5 = -3$
- *Number f terms, n =?*
- *Sum of n terms,* S_{10} =?

We know that *n*th term of this A.P.

$l = a + (n - 1)d$

$-230 = -5 + (n - 1)(-3)$

$-225 = (n - 1)(-3)$

$(n - 1) = 75$

Hence, $n = 76$

We know that sum of n terms of AP,

$$S_n = \frac{n}{2}[a + l]$$

$$= \frac{76}{2}[(-5) + (-230)]$$

$$= 38(-235)$$

$$= -8930$$

Question 3:
In an AP
(i) Given $a = 5$, $d = 3$, $an = 50$, find n and Sn.

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}.

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(v) Given $d = 5$, $S_9 = 75$, find a and a_9.

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and an.

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.

(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.

(x)Given $l = 28$, $S = 144$ and there are total 9 terms. Find a.

Solution 3:

(i) Given that,

- *First term*, $a = 5$
- Common difference, $d = 3$
- Nth term of AP series, $a_n = 50$
- Number of terms, n=?
- *Su of n terms*, $S_n = ?$

As $a_n = a + (n - 1)d$,
$\therefore 50 = 5 + (n - 1)3$
$45 = (n - 1)3$
$15 = n - 1$
$n = 16$
$$S_n = \frac{n}{2}[a + a_n]$$

$$S_{16} = \frac{16}{2}[5+50]$$
$$= 8 \times 55$$
$$= 440$$

(ii) Given that,

- *First term,* $a = 7$
- *Number of terms,* $n = 13$
- *13^{th} term of AP,* $a_{13} = 35$
- Common difference, d=?
- *Sum of n terms,* $S_{13} = ?$

We know that nth term of the AP series, $a_n = a + (n - 1)\, d$,

$\therefore a_{13} = a + (13 - 1)\, d$ *(By Substituting)*

$35 = 7 + 12\, d$

$35 - 7 = 12d$

$28 = 12d$

$$d = \frac{7}{3}$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_{13} = \frac{n}{2}[a + a_{13}]$$ *(By Substituting)*

$$= \frac{13}{2}[7+35]$$

$$= \frac{13 \times 42}{2} = 13 \times 21$$

$$= 273$$

(iii) Given that,

- *First term,* $a = ?$
- *Common difference,* $d = 3$
- *12^{th} term of AP series,* $a_{12} = 37$
- Number of terms, n=12
- First term, a=?
- *Sum of n terms,* $S_{12} = ?$

We know that nth term of the AP series, $a_n = a + (n - 1)d$,

$a_{12} = a + (12 - 1)3$

$37 = a + 33$

$a = 4$

$S_n = \frac{n}{2}[a + a_n]$

$S_n = \frac{12}{2}[4 + 37]$

$S_n = 6(41)$

$S_n = 246$

(iv) Given that,

- *Third Term,* $a_3 = 15$
- Number of terms, n=10
- *Sum of n terms,* $S_{10} = 125$
- *Common difference,* $d = ?$
- *10th Term,* $a_{10} = ?$

We know that nth term of the AP series, $a_n = a + (n - 1)d$,

$a_3 = a + (3 - 1)d$

$15 = a + 2d$---------------------*Equation* (i)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2a + (10-1)d]$

$125 = 5(2a + 9d)$

$25 = 2a + 9d$

On multiplying equation (1) by 2, we obtain

$30 = 2a + 4d$-----------*Equation* (iii)

On subtracting equation (iii) from Equation (ii), we obtain

$-5 = 5d$

$d = -1$

From equation (i),

$15 = a + 2(-1)$

$15 = a - 2$

$a = 17$

$a_{10} = a + (10 - 1)d$

$a_{10} = 17 + (9)(-1)$

$a_{10} = 17 - 9 = 8$

(v) Given that,

- *Common Difference,* $d = 5$
- Number of terms, n=9
- *Sum of n terms,* $S_{10} = 75$

- *First term, a* = ?
- 9^{th} *term,* a_9 = ?

As ,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{9}{2}\left[2a + (9-1)5\right]$$

$$75 = \frac{9}{2}(2a + 40)$$

25 = 3(a + 20)
25 = 3a + 60
3a = 25 – 60

$$a = \frac{-35}{3}$$

We know that nth term of the AP series, $a_n = a + (n - 1)d$
$a_9 = a + (9 - 1)(5)$

$$= \frac{-35}{3} + 8(5)$$

$$= \frac{-35}{3} + 40$$

$$= \frac{-35 + 120}{3} = \frac{85}{3}$$

(vi) Given that,

- *First term, a* = 2
- *Common difference, d* = 8
- *Sum of n terms,* $S_n = 90$
- *nth term of AP,* a_n = ?
- number of terms, n=?

As , $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$$90 = \frac{n}{2}\left[4 + (n-1)8\right]$$

$90 = n\,[2 + (n - 1)4]$
$90 = n\,[2 + 4n - 4]$
$90 = n\,(4n - 2) = 4n^2 - 2n$
$4n^2 - 2n - 90 = 0$
$4n^2 - 20n + 18n - 90 = 0$
$4n\,(n - 5) + 18\,(n - 5) = 0$
$(n - 5)\,(4n + 18) = 0$

Either $n - 5 = 0$ or $4n + 18 = 0$

$n = 5$ or $= -\frac{18}{4} = \frac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, $n = 5$

$a_n = a + (n - 1)d$

$a_5 = 2 + (5 - 1)8$

$= 2 + (4)\,(8)$

$= 2 + 32 = 34$

(vii) Given that,

- *First term, a* = 8
- *Nth term of AP,* $a_n = 62$
- *Sum of n terms,* $S_n = 210$
- *Common difference, d* = ?
- Number of terms, n=?

$$S_n = \frac{n}{2}[a + a_n]$$

$$210 = \frac{n}{2}[8 + 62] \qquad \text{(By Substituiting)}$$

$$210 = \frac{n}{2}(70)$$

$n = 6$

We know that nth term of the AP series, $a_n = a + (n - 1)d$

$62 = 8 + (6 - 1)d$

$62 - 8 = 5d$ (By Substituiting)

$54 = 5d$

$$d = \frac{54}{5}$$

(viii) Given that,

- *Common difference, d* = 2
- *Nth terms of AP,* $a_n = 4$
- *Sum of n terms,* $S_n = -14$
- *First term, a* = ?
- Number of terms, n=?

We know that nth term of AP series, $a_n = a + (n - 1)d$

$4 = a + (n - 1)2$

$4 = a + 2n - 2$(By Substituting)

$a + 2n = 6$

$a = 6 - 2n$-------------*Equation* (i)

$$S_n = \frac{n}{2}[a + a_n]$$

$$-41 = \frac{n}{2}[a + 4]$$

$-28 = n(a + 4)$
$-28 = n(6 - 2n + 4)$ {From equation (i)}
$-28 = n(-2n + 10)$
$-28 = -2n^2 + 10n$
$2n^2 - 10n - 28 = 0$
$n^2 - 5n - 14 = 0$
$n^2 - 7n + 2n - 14 = 0$
$n(n - 7) + 2(n - 7) = 0$
$(n - 7)(n + 2) = 0$
Either $n - 7 = 0$ or $n + 2 = 0$
$n = 7$ or $n = -2$
However, n can neither be negative nor fractional.
Therefore, $n = 7$
From equation (i), we obtain
$a = 6 - 2n$
$a = 6 - 2(7)$
$= 6 - 14$
$= -8$

(ix)Given that,

- *First term,* $a = 3$
- Number of terms, n=8
- *Sum of n terms,* $S_n = 192$
- *Common difference,* $d = ?$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$192 = \frac{8}{2}[2 \times 3 + (8 - 1)d]$$

$192 = 4[6 + 7d]$
$48 = 6 + 7d$
$42 = 7d$
$d = 6$

(x) Given that,

- *Last term,* $l = a_n = 28$
- Number of terms, n=9

- *Sum of n terms, S_n =144*
- *First term, a* = ?

$$S_n = \frac{n}{2}(a+1)$$

$$144 = \frac{9}{2}(a+28)$$

(16) × (2) = *a* + 28
32 = *a* + 28
a = 4

Question 4:
How many terms of the AP. 9, 17, 25 … must be taken to give a sum of 636?

Solution 4:
Given,

- First Term, a= 9
- Common Difference, $d = a_2 - a_1 = 17 - 9 = 8$
- Sum of n terms S_n =*636*
- *Number of terms, n= ?*

We know that sum of n terms of AP

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$636 = \frac{n}{2}[2 \times a + (n-1)8]$$

$$636 = \frac{n}{2}[18 + (n-1)8]$$

636 = *n* [9 + 4*n* − 4]
636 = *n* (4*n* + 5)
$4n^2 + 5n - 636 = 0$
$4n^2 + 53n - 48n - 636 = 0$
n (4*n* + 53) − 12 (4*n* + 53) = 0
(4*n* + 53) (*n* − 12) = 0
Either 4*n* + 53 = 0 or *n* − 12 = 0

$n = \frac{-53}{4}$ Or n = 12

n cannot be $\frac{-53}{4}$.As the number of terms can neither be negative nor fractional, therefore, *n* = 12 only.

Question 5:
The first term of an AP is 5, the last term is 45 and the sum is 400.
Find the number of terms and the common difference.

Solution 5:
Given that,

- *First term,* $a = 5$
- *Last term,* $l = a_n = 45$
- *Sum of n terms,* $S_n = 400$
- *Number of terms,* $n = ?$
- *Common difference,* $d = ?$

We know that sum of n terms,

$$S_n = \frac{n}{2}(a+1)$$

$$400 = \frac{n}{2}(5+45) \text{ (By Substituitng)}$$

$$400 = \frac{n}{2}(50)$$

$n = 16$

$l = a + (n - 1)\,d$

$45 = 5 + (16 - 1)\,d$

$40 = 15d$

$$d = \frac{40}{15} = \frac{8}{3}$$

Question 6:
The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution 6:
Given that,

- *First term,* $a = 17$
- *Last term,* $l = 350$
- *Common difference,* $d = 9$
- *Number of terms,* $n = ?$
- *Sum of n terms,* $S_n = ?$

We know that nth term of AP, $l = a + (n - 1)\,d$

$350 = 17 + (n - 1)9$
$333 = (n - 1)9$(By Substituting)
$(n - 1) = 37$
$n = 38$
Sum of n terms of AP,

$$S_n = \frac{n}{2}(a + 1)$$

$$\Rightarrow S_n = \frac{38}{2}(17 + 350) = 19(367) = 6973$$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Question 7:

Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Solution 7:

Given,

- *Common Difference,* $d = 7$
- *22^{nd} term,* $a_{22} = 149$
- *Sum of n terms,* $S_{22} = ?$
- $a = ?$

We know that nth term of AP, $a_n = a + (n - 1)d$
$a_{22} = a + (22 - 1)d$
$149 = a + 21 \times 7$ (By Substituting)
$149 = a + 147$
$a = 2$

$$S_n = \frac{n}{2}(a + a_n)$$

$$= \frac{22}{2}(2 + 149)$$

$$= 11(151) = 1661$$

Question 8:

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution 8:

Given that,

- *First term, a=?*
- *Second term, a_2* = 14
- *Third Term, a_3* = 18
- *Common difference, $d = a_3 - a_2$* = 18 − 14 = 4
- *Sum of n terms,   =?*

We know that second term, $a_2 = a + d$

$14 = a + 4$

$a = 10$

Sum of nth term of series,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{51} = \frac{51}{2}\left[2 \times 10 + (51-1)4\right]$$

$$= \frac{51}{2}\left[20 + (50)(4)\right]$$

$$= \frac{51(220)}{2} = 51(110)$$

= 5610

Question 9:

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first *n* terms.

Solution 9:

Given that,

- *Sum on 7 terms, S_7* = 49
- Sum of 17 terms, S_{17} = 289
- Sum of n terms, S_n = ?
- Number of terms, n =?
- Common difference, d=?
- First term, a=?

We know that sum of n term of AP is ,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{51} = \frac{7}{2}\left[2a + (7-1)d\right]$$

$$49 = \frac{7}{2}\left[2a + 6d\right]$$

$$7 = (a + 3d)$$

$$a + 3d = 7 \ \ (i)$$

Similarly, $S_{17} = \frac{17}{2}\left[2a + (17-1)d\right]$

$$289 = \frac{17}{2}\left[2a + 16d\right]$$

$17 = (a + 8d)$

$a + 8d = 17$ (ii)

Subtracting equation (i) from equation (ii),

$5d = 10$

$d = 2$

From equation (i),

$a + 3(2) = 7$

$a + 6 = 7$

$a = 1$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2(1) + (n-1)(2)\right]$$

$$= \frac{n}{2}(2 + 2n - 2)$$

$$= \frac{n}{2}(2n)$$

$$= n^2$$

Question 10:

Show that $a1$, $a2$ … , an , … form an AP where an is defined as below

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Solution 10:

(i)Given,

- Nth term of AP, $a_n = 3 + 4n$

- *Common difference, d=?*
- *First term, a=?*
- *Sum of 15 terms,* $S_{n=?}$

$a_1 = 3 + 4(1) = 7$
$a_2 = 3 + 4(2) = 3 + 8 = 11$
$a_3 = 3 + 4(3) = 3 + 12 = 15$
$a_4 = 3 + 4(4) = 3 + 16 = 19$
It can be observed that
$a_2 - a_1 = 11 - 7 = 4$
$a_3 - a_2 = 15 - 11 = 4$
$a_4 - a_3 = 19 - 15 = 4$
i.e., a_{n+1} *and* a_n is same every time.
Therefore, this is an AP with common difference as 4 and first term as 7.
Sum of n terms,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(7) + (15-1)4\right]$$

$$= \frac{15}{2}((14) + 56)$$

$$= \frac{15}{2}(70)$$

$$= 15 \times 35$$

$$= 525$$

(ii)Given,

- *Nth term of AP,* $a_n = 9 - 5n$
- *Common difference, d=?*
- *First term, a=?*

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$
$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$
$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$
$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$
It can be observed that
$a_2 - a_1 = -1 - 4 = -5$
$a_3 - a_2 = -6 - (-1) = -5$
$a_4 - a_3 = -11 - (-6) = -5$
i.e., $a_{k+1} - a_k$ is same every time.
Therefore, this is an A.P. with common difference as −5 and first term as 4.
Sum of n terms of AP,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$S_{15} = \frac{15}{2}\left[2(4)+(15-1)(-5)\right]$$
$$= \frac{15}{2}(8+14(-5))$$
$$= \frac{15}{2}(8-70)$$
$$= \frac{15}{2}(-62) = 15(-31)$$
$$= -465$$

Question 11:

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the10th and the nth terms.

Solution 11:

Given,

- *Sum of first n terms,* $S_n = 4n - n^2$
- Sum of First term, $a = S_1 = ?$
- Sum of first two terms = $S_2 = ?$
- Second term, $a_2 = ?$
- *Common Difference,* $d = a_2 - a = 1 - 3 = -2$
- Third term, $a_3 = ?$
- 10^{th} term, $a_{10} = ?$
- Nth term, $a_n = ?$

Sum of First term, $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$
Sum of first two terms = $S_2 = ?= 4(2) - (2)^2 = 8 - 4 = 4$
Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$
We know that sum of n terms, $a_n = a + (n - 1)d$
$= 3 + (n - 1)(-2)$
$= 3 - 2n + 2$
$= 5 - 2n$
Therefore, $a_3 = 5 - 2(3) = 5 - 6 = -1$
$a_{10} = 5 - 2(10) = 5 - 20 = -15$

Hence, the sum of first two terms is 4.
The second term is 1.

3rd, 10th, and nth terms are −1, −15, and 5 − 2n respectively.

Question 12:
Find the sum of first 40 positive integers divisible by 6.

Solution 12:
The positive integers that are divisible by 6 are 6, 12, 18, 24 …
It can be observed that these are making an A.P.
Hence,

- first term, a= 6
- common difference ,d =6.
- *Sum of 40 terms,* S_{40} *=?*

As we know that Sum of n terms,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[2(6) + (40-1)6\right]$$

= 20[12 + (39) (6)]
= 20(12 + 234)
= 20 × 246
= 4920

Question 13:
Find the sum of first 15 multiples of 8.

Solution 13:
The multiples of 8 are
8, 16, 24, 32…
These are in an A.P.,
Hence,

- *First term, a* = 8
- *Common difference, d* = 8
- *Sum of 15 terms,* S_{15} =?

Therefore,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{15}{2}\left[2(8) + (15-1)8\right]$$

$= \frac{15}{2}(16 + 14(8))$

$= \frac{15}{2}(16 - 112)$

$= \frac{15(128)}{2} = 15 \times 64$

$= 960$

= 960

Question 14:

Find the sum of the odd numbers between 0 and 50.

Solution 14:

The odd numbers between 0 and 50 are
1, 3, 5, 7, 9 … 49
Therefore, it can be observed that these odd numbers are in an A.P.

- *First term, a* = 1
- *Common difference, d* = 2
- *Last term, l* = 49
- *Sum of odd numbers between 0 and 50, S_n* =?

We know that nth term of AP, $l = a + (n - 1)d$

$49 = 1 + (n - 1)2$

$48 = 2(n - 1)$

$n - 1 = 24$

$n = 25$

We know that sum of n terms of AP,

$S_n = \frac{n}{2}(a + 1)$

$S_{25} = \frac{25}{2}(1 + 49)$

$= \frac{25(50)}{2} = (25)(25)$

$= 625$

Question 15:

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Solution 15:

By observation that these penalties are in an A.P. having first term as 200 and common difference as 50.

$a = 200$

$d = 50$

Penalty that has to be paid if he has delayed the work by 30 days = S_{30}

$= \frac{30}{2}[2(200)+(30-1)50]$

$= 15[400+1450]$

$= 15(1850)$

$= 27750$

Therefore, the contractor has to pay Rs 27750 as penalty.

Question 16:

A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Solution 16:

- Let the cost of 1st prize be P.
- Cost of 2nd prize = $P - 20$
- And cost of 3rd prize = $P - 40$

By observation that the cost of these prizes are in an A.P. having common difference as −20 and first term as P.

$a = P$

$d = -20$

Given that, $S_7 = 700$

$\frac{7}{2}[2a+(7-1)d] = 700$

$\frac{[2a+(6)(-20)]}{2} = 100$

$a + 3(-20) = 100$

$a - 60 = 100$

$a = 160$

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

Question 17:

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Solution 17:

It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 5………………..12

- First term, $a = 1$
- Common difference, $d = 2 - 1 = 1$

We know that sum of n terms of AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}(2(1) + (12-1)(1))$$

= 6 (2 + 11)

= 6 (13)

= 78

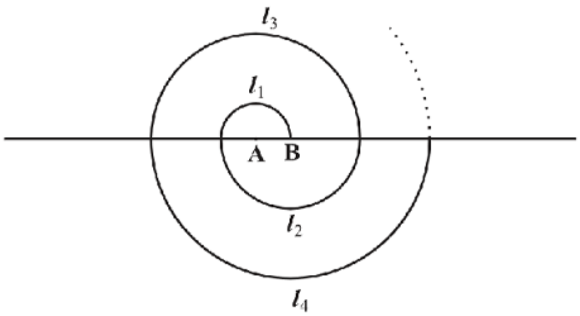
Therefore, number of trees planted by 1 section of the classes = 78

Number of trees planted by 3 sections of the classes = 3 × 78 = 234

Therefore, 234 trees will be planted by the students.

Question 18:

A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ……… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? $\left[\text{Take } \pi = \frac{22}{7}\right]$



Solution 18:

Semi-perimeter of circle = πr

$I_1 = \pi(0.5) = \frac{\pi}{2}$ cm

$I_2 = \pi(1) = \pi$ cm

$I_3 = \pi(1.5) = = \frac{3\pi}{2}$ cm

Therefore, I_1, I_2, I_3 ,i.e. the lengths of the semi-circles are in an A.P.,

$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ …….

$a = \frac{\pi}{2}$

$d = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

$S_{13} = ?$

We know that the sum of *n* terms of an a A.P. is given by

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{13}{2}\left(2\left(\frac{\pi}{2}\right) + (13-1)\left(\frac{\pi}{2}\right)\right)$

$= \frac{13}{2}\left[\pi + \frac{12\pi}{2}\right]$

$= \left(\frac{13}{2}\right)(7\pi)$

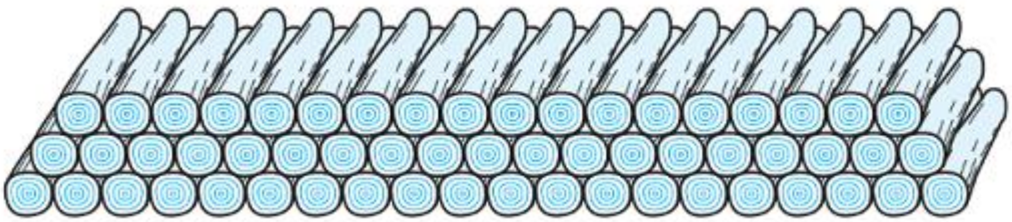
$= \frac{91\pi}{2}$

$= \frac{91 \times 22}{2 \times 7} = 13 \times 11$

$= 143$

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

Question 19:
200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Solution 19:
It can be observed that the numbers of logs in rows are in an A.P.
20, 19, 18…
For this A.P.,

- *First term, a* = 20
- *Common Difference*, $d = a_2 - a_1 = 19 - 20 = -1$

Let a total of 200 logs be placed in n rows.

- $S_n = 200$

We know that sum of n terms of AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$200 = \frac{n}{2}\left(2(20) + (n-1)(-1)\right)$$

$400 = n(40 - n + 1)$
$400 = n(41 - n)$
$400 = 41n - n2$
$n^2 - 41n + 400 = 0$
$n^2 - 16n - 25n + 400 = 0$
$n(n - 16) - 25(n - 16) = 0$
$(n - 16)(n - 25) = 0$
Either $(n - 16) = 0$ or $n - 25 = 0$
$n = 16$ or $n = 25$

$\Rightarrow$ n = 16 or 25

Here the common difference is negative.

The terms go on diminishing and 21st term becomes zero. All terms after 21st term are negative. These negative terms when added to positive terms from 17th term to 20th term, cancel out each other and the sum remains the same.

Thus n = 25 is not valid for this problem. So we take n = 16.

Thus, 200 logs are placed in 16 rows.

Number of logs in the 16th row

$= a_{16}$

$= a + 15d$

$= 20 + 15(-1)$

$= 20 - 15 = 5$

$a_n = a + (n - 1)d$

$a_{16} = 20 + (16 - 1)(-1)$

$a_{16} = 20 - 15$

$a_{16} = 5$

Similarly,

$a_{25} = 20 + (25 - 1)(-1)$

$a_{25} = 20 - 24$

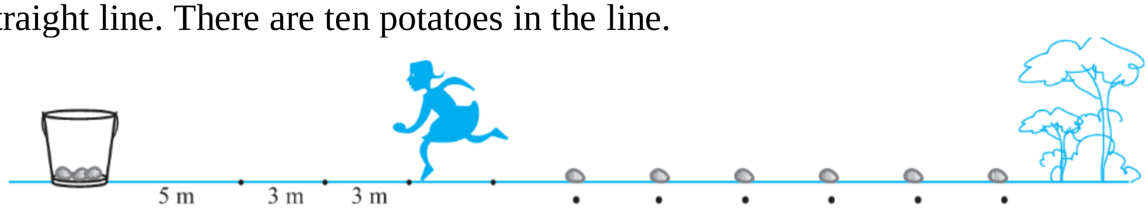
$= -4$

Clearly, the number of logs in 16th row is 5. However, the number of logs in 25th row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

Question 20:

In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.

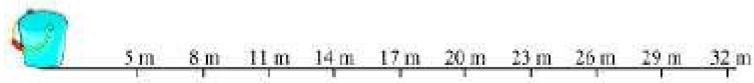


A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

Solution 20:

The distances of potatoes are as follows.
5, 8, 11, 14…



It can be observed that these distances are in A.P.

- *First term, a* = 5
- *Common difference, d* = 8 − 5 = 3

We know that the sum of n terms,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}(2(5) + (10-1)3)$$

= 5[10 + 9 × 3]
= 5(10 + 27) = 5(37)
= 185
As every time she has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it.
Therefore, total distance that the competitor will run = 2 × 185
= 370 m

Alternatively,

The distances of potatoes from the bucket are 5, 8, 11, 14…
Distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept. Therefore, distances to be run are

10, 16, 22, 28, 34,……….
a = 10
d = 16 − 10 = 6
S_{10} =?

$$S_{10} = \frac{10}{2}(2 \times 10 + (10-1)6)$$

= 5[20 + 54]
= 5 (74)
= 370

Therefore, the competitor will run a total distance of 370 m.

EXERCISE NO: 5.4

Question 1:
Which term of the A.P. 121, 117, 113 … is its first negative term?
[Hint: Find n for $a_n < 0$]

Solution 1:
Given:

- A.P. Series: 121, 117, 113 …
- First Term, $a = 121$
- Common Difference, $d = 117 - 121 = -4$
- The first negative term of this A.P. Series =?

We know that n^{th} term of AP, $a_n = a + (n - 1) d$
By substituting the above values

$$a_n = 121 + (n - 1)(-4)$$
$$= 121 - 4n + 4$$
$$= 125 - 4n$$

For the first negative term of this A.P,

Therefore, $a_n < 0$

$125 - 4n < 0$

$\Rightarrow 125 < 4n$

$\Rightarrow n > \frac{125}{4}$

$\Rightarrow n > 31.25$

Therefore, 32^{nd} term will be the first negative term of this A.P.

Question 2:
The sum of the third and the seventh terms of an A.P is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.

Solution 2:
Given:

- $a_3 + a7 = 6$ --------------Equation (1)
- $(a_3) \times (a_7) = 8$ ---------- Equation (2)

We know that n^{th} *term of AP Series,*
$a_n = a + (n - 1) d$

Third Term, $a_3 = a + (3 - 1) d$
Hence, $a_3 = a + 2d$ ----------------Equation (3)
Seventh Term, $a_7 = a + 6d$ ----------------Equation (4)

Using Equation (3) and Equation (4) in Equation (1) ,

$(a + 2d) + (a + 6d) = 6$
$2a + 8d = 6$
$a + 4d = 3$
$a = 3 - 4d$ ----------Equation(5)

Using Equation (3) and Equation (4) in Equation(2) ,
$(a + 2d) \times (a + 6d) = 8$

Substituting the value of Equation(5) in above,

$$(3-4d+2d)\times(3-4d+6d)=8$$
$$(3-2d)\times(3+2d)=8$$
$$9-4d^2=8$$
$$4d^2=9-8=1$$
$$d^2=\frac{1}{4}$$
$$d=\pm\frac{1}{2}$$
$$d=\frac{1}{2}\text{ or }-\frac{1}{2}$$

Hence by substituting both the values of d,

$$\left(\text{When d is }\frac{1}{2}\right)$$
$$a=3-4d$$
$$a=3-4\left(\frac{1}{2}\right)$$
$$=3-2=1$$
$$\left(\text{When d is }-\frac{1}{2}\right)$$
$$a=3-4\left(-\frac{1}{2}\right)$$
$$a=3+2=5$$

We know that Sum of Nth term of AP Series,

$S_n = \frac{n}{2}\left[2a(n-1)d\right]$

$\left(\text{When a is 1 and d is } \frac{1}{2}\right)$

$S_{16} = \frac{16}{2}\left[2(1)+(16-1)\left(\frac{1}{2}\right)\right]$

$= 8\left[2+\frac{15}{2}\right]$

$= 4(19) = 76$

$\left(\text{When a is 5 and d is } -\frac{1}{2}\right)$

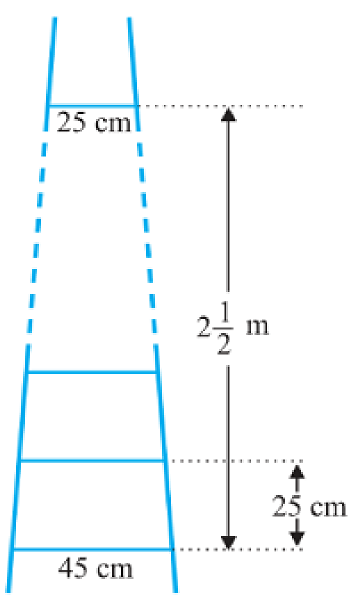
$8\left[10+(15)\left(-\frac{1}{2}\right)\right]$

$= 8\left(\frac{5}{2}\right)$

$= 20$

Question 3:

A ladder has rungs 25 cm apart. (See figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs? [Hint: number of rungs $= \frac{250}{25}$]



Solution 3:

Given:

- Distance between the rungs= 25cm
- Distance between the top and bottom rungs = $2\frac{1}{2}$ m = $2\frac{1}{2}\times 100$ cm

∴ Total number of rungs $=\frac{2\frac{1}{2}\times 100}{25}+1=\frac{250}{25}+1=11$

From the given Figure, we can observe that the lengths of the rungs decrease uniformly, hence we can conclude that they will be in an AP

The length of the wood required for the rungs equals the sum of all the terms of this A.P.

- First term, a = 45
- Last term, l = 25
- *No of terms*, n = 11

Hence Sum of nth Series,

$$S_n=\frac{n}{2}(a+l)$$

$$\therefore S_{10}\frac{11}{2}(45+25)=\frac{11}{2}(70)=385 \text{ cm}$$

Therefore, the length of the wood required for the rungs is 385 cm.

Question 4:

The houses of a row are number consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of the houses preceding the house numbered x is equal to the sum of the number of houses following it.

Find this value of x.

[Hint $Sx - 1 = S49 - Sx$]

Solution 4:

Given :

Number of houses was 1,2,3,……………..49

By Observation, the number of houses are in an A.P.

Hence

- *First Term, a= 1*
- *Common Difference, d =1*

Let us assume that the number of x^{th} house can be expressed as below:

We know that, Sum of n terms in an A.P. = $\frac{n}{2}[2a+(n-1)d]$

Sum of number of houses preceding x^{th} house = $S_x - 1$

$= \frac{(x-1)}{2}\left[2a + (x-1-1)d\right]$

$= \frac{x-1}{2}\left[2(1) + (x-2)(1)\right]$

$= \frac{x-1}{2}\left[2 + x - 2\right]$

$= \frac{(x)(x-1)}{2}$

By the given we know that, Sum of number of houses following xth house = $S_{49} - S_x$

$= \frac{49}{2}\left(2(1) + (49-1)(1)\right) - \frac{x}{2}\left[2)1 + (x-1)(1)\right]$

$= \frac{49}{2}(2 + 49 - 1) - \frac{x}{2}(2 + x - 1)$

$= \left(\frac{49}{2}\right)(50) - \frac{x}{2}(x+1)$

$= 25(49) - \frac{x(x+1)}{2}$

It is given that these sums are equal to each other.

$\frac{x(x-1)}{2} = 25(49) - x\left(\frac{x+1}{2}\right)$

$\frac{x^2}{2} - \frac{x}{2} = 1225 - \frac{x^2}{2} - \frac{x}{2}$

$x^2 = 1225$

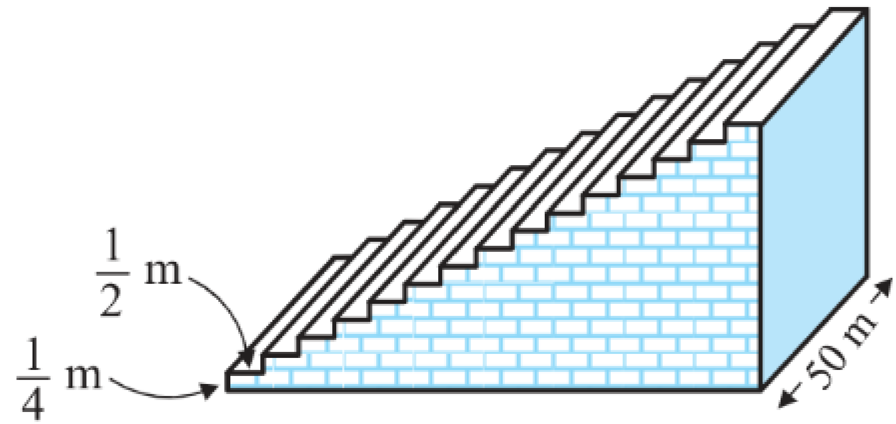
$x = \pm 35$

As the number of houses cannot be a negative number we consider number of houses, x = 35.

Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

Question 5:

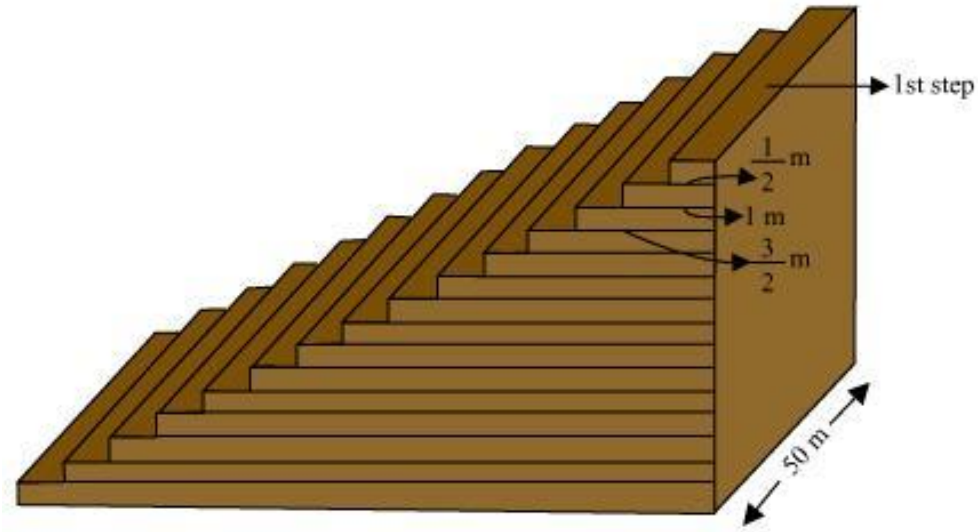
A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (See figure) calculate the total volume of concrete required to build the terrace.



Solution 5:

Given:

- From the figure, it can be observed that1st step is $\frac{1}{2}$ m wide,2nd step is 1 m wide,3rd step is $\frac{3}{2}$ m wide. Therefore, the width of each step is increasing by $\frac{1}{2}$ m each time height $\frac{1}{4}$ m
- length 50 m remains the same. Widths of these steps are $\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots\ldots$



Volume of Step can be considered as Volume of Cuboid= Length X Breadth X Height

Volume of concrete in 1st step $=\frac{1}{4}\times\frac{1}{2}\times 50=\frac{25}{4}$

Volume of concrete in 2nd step $=\frac{1}{4}\times 1\times 50=\frac{25}{2}$

Volume of concrete in 3rd step $=\frac{1}{4}\times\frac{3}{2}\times 50=\frac{75}{4}$

It can be observed that the volumes of concrete in these steps are in An A.P.

$$\frac{25}{4},\frac{25}{2},\frac{75}{4},.....$$

First Term, $a=\frac{25}{4}$

$$d=\frac{25}{2}-\frac{25}{4}=\frac{25}{4}$$

Sum of nth term,

and $S_n=\frac{n}{2}[2a+(n-1)d]$

$$S_{15}=\frac{15}{2}\left(2\left(\frac{25}{4}\right)+(15-1)\frac{25}{4}\right)$$

$$=\frac{15}{2}\left[\frac{25}{2}+\frac{(14)25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{175}{2}\right]$$

$$= \frac{15}{2}(100) = 750$$

Volume of concrete required to build the terrace is 750 m^3.
